EXPERIMENTAL TEST AND APPLICATION OF A 2-D FINITE ELEMENT CALCULATION FOR WHISPERING GALLERY SAPPHIRE RESONATORS*

David **G. Santiago,** Rabi T. Wang and G. John Dick
Jet Propulsion Laboratory
California institute. of Technology
Pasadena, CA 91109

and

Roberto A. Osegueda, Joseph H. Pierluissi, Luis M. Gil,
Arturo Revilla and Gustavo J. Villalva
The University of Texas at El Paso
College of Engineering
El Paso, TX 79968

Abstract

This paper discusses the demonstrated accuracy and utility of a 2-D finite element methodology for whispering gallery mode sapphire resonators. The mode solutions obtained by the software compared with experimental results for a wheel-shaped sapphire resonator give an error in mode frequency of less than .55%. We also show parts per million agreement with analytical solutions for simple geometries such as an empty coaxial resonator. The CYRES 2D FEM software package developed at The University of Texas at El Paso has proven invaluable for the analysis and identification of modes and mode families for resonators of various geometries. The software also shows promise as a tool for optimization of new resonator designs. Current uses include design of optimum sized dielectric resonators for minimized wall losses, and new resonator geometries for temperature compensated resonators. The operational characteristics of the software and the general methodology for use of the software as a laboratory and design tool are discussed.

Introduction

Recently, whispering gallery mode (WGM) resonators consisting of a sapphire dielectric element in a metallic container, as in Figure 1, have made possible new capabilities for microwave oscillator phase noise and frequency stability [1,2,3,4]. With high azimuthal mode numbers, these resonators isolate radio-frequency energy to the dielectric element and away from the metallic container, thus providing extraordinarily high quality factors (Q's >10 million). Design and analysis of such a resonator with a certain frequency and Q requires a finite element method (FEM) model with higher resolution and accuracy than any previously available.

Modeling Challenges

The characteristics and exceptional properties of sapphire whispering gallery mode resonators make them particularly challenging to model using finite element analysis. The finite element model must allow for widely disparate field magnitudes in order to resolve the hybrid fields that exist in WGM resonators. The electromagnetic fields of whispering gallery modes are well confined to the sapphire element [5]. This creates areas of high field strength in the sapphire and weak field areas outside the sapphire, presenting a substantial burden for any calculational technique. Accurate modeling of these critical areas requires a high FEM node density to provide sufficient resolution. Because even small wall losses are important for preserving the very high Q's, accurate calculation of the low field strengths at the container wall by the FEM model is essential. Furthermore, the coupling ports for our WGM resonator are located in the evanescent region, and so an accurate determination of these weak field strengths is necessary to determine resonator coupling.

FEM software must eliminate spurious modes without losing any of the many modes found in a WGM resonator. The anisotropy of the sapphire dielectric must also be considered for proper FEM modeling. When the z-axis is aligned with the sapphire crystal c-axis, the resonator is anisotropic in two dimensions with $\varepsilon_z \approx 11$ and $\varepsilon_r = \varepsilon_\phi \approx 9$. A three-dimensional

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FEM allowing full treatment of sapphire's anisotropic dielectric constant, would requite. such a large. number of elements as to be impractical. Approximate analytical methods are useful for some geometries, but a new approach would be required for every change in geometry. A two-dimensional finite element approach, however, allows easy treatment of any cylindrically symmetric resonator geometry.

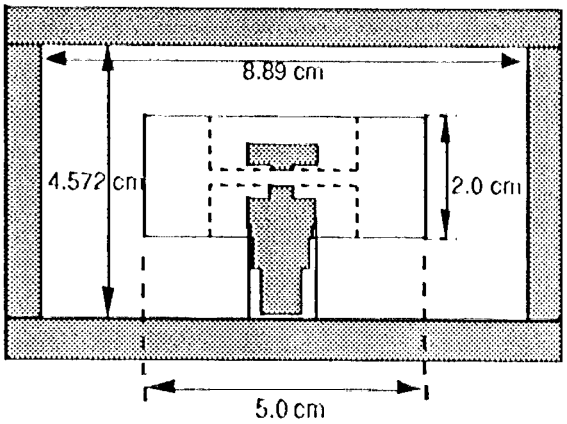

Figure 1. Sapphire Resonator in Containment Can

CYRES 2D

Because. the dielectric constant for sapphire shows cylindrical symmetry, a two dimensional treatment is allowed for the important case, where its crystal c axis is aligned with a physical axis of axisymmetry. While ruling out most anisotropic dielectric configurations, this approach makes possible. the first two-dimensional finite element treatment for sapphire "whispering gallery" resonators. For the different modes of a given circularly symmetric geometry, the CYRES 2D finite element package, under development at the University of Texas at El Paso, allows determination of resonant frequencies and visualization of the electromagnetic fields [6,7].

A finite. element mesh of rectilinear or curved elements defines the resonator in the r-z plane. Maxwell's equations for the anisotropic case are solved for the cross section. To complete the 3D solution, the method assumes a sinusoidal dependence in azimuth. The 2D solution is projected around [he. resonator by multiplication with the azimuthal sinusoidal dependence. These field solutions provide the resonant frequencies and cavity Q's(determined by will losses) of each mode. A penalty term in the. vector Helmholtz equations is used to suppress spurious solutions from appearing in the final list of resonant modes.

Validation

During development the FEM solutions were compared with several analytically determined solutions to test the software. Upon delivery to JPL from UTEP the. software underwent additional validation testing to determine the FEM model's accuracy limits for different mesh densities. Figure 2 shows the fractional frequency

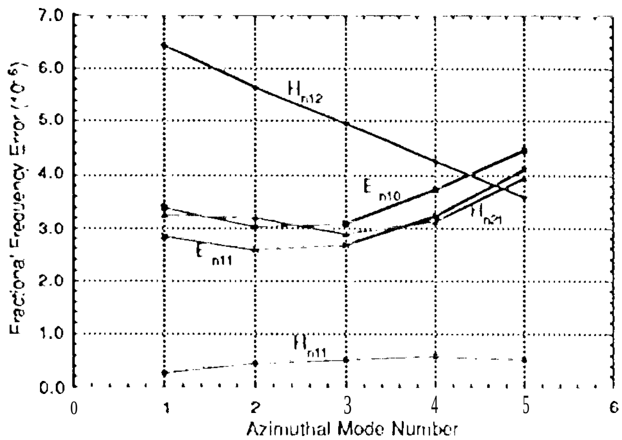

Figure ?. FEM Data Compared to Analytic Solution for an Empty Coaxial Resonator

error of the CYRES mode solutions for an empty coaxial resonator compared 10 the analytic exact solution. As seen in the figure, the largest error is about 6.5 parts per million.

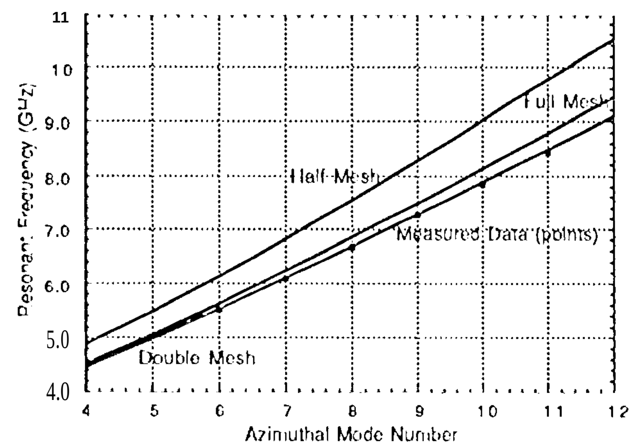

Figure 3. FEM calculated WGM frequencies for different mesh densities compared to laboratory measurements

High mesh densities for FEM calculations produce higher accuracy results with greater resolution. The benefits of superior accuracy arc. achieved at the expense of computational complexity and speed. Typically, our chosen quadrilateral element "full mesh" contains about 200 nodes. Calculation of the resonant mode solutions is also performed at half this mesh density (half mesh) and double [his density (double mesh), which respectively reduce and increase the number of nodes try a factor of four. Figure 3 shows the half, full and double mesh density FEM calculated resonant frequencies for the WGM resonator of Figure 1. The points which fall along the double mesh line arc the laboratory measurements of the resonant frequencies.

By choosing a full mesh density of sufficient accuracy we can use the half mesh and double mesh calculations to extrapolate FEM solutions to an infinite mesh density, thereby providing excellent modeling accuracy for a modest computational expense. Figure 4 shows the fractional frequency difference Of the three mesh densities and the measured data from the

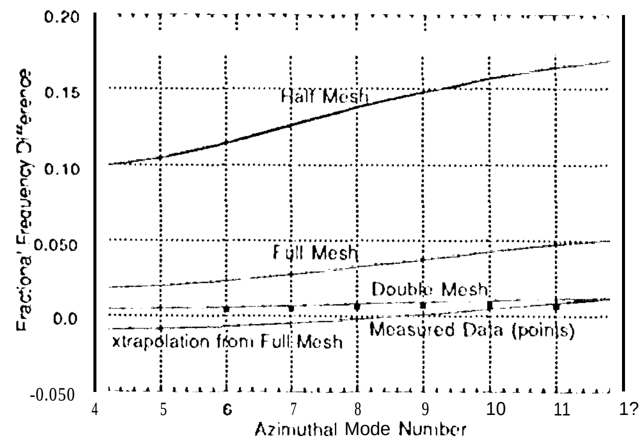

Figure 4. Difference comparison of FEM calculated frequencies for different mesh densities to laboratory measurements (zero equals extrapolation to infinite mesh)

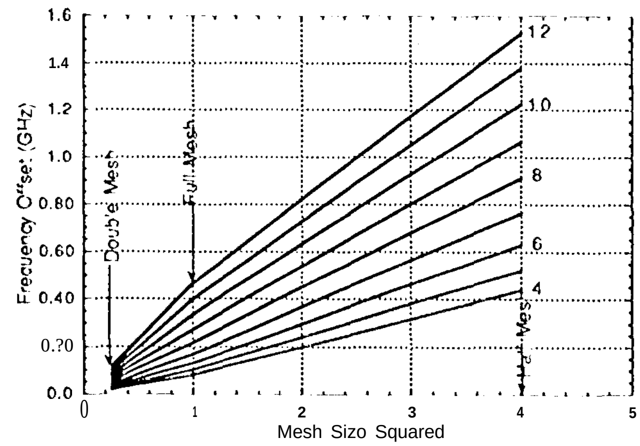

Figure 5. Calculated FEM Frequency offset vs. Mesh Density

extrapolation to infinite. mesh density. A quadratic extrapolation to infinite. mesh based on full and double meshes defines the zero on the. y-axis. The measured data points arc also plotted and fall no more. than 0.5% above. the. extrapolated solution. Also shown is the. "Extrapolation from Full Mesh" that uses only the half and full mesh solutions to extrapolate to an infinite mesh solution.

Higher order modes intrinsically require higher resolution calculations to provide accuracy equivalent to lower order mode solutions. Figure 5 shows the offset from the infinite mesh mode frequency solutions of the different mesh densities for different order modes. The nearly straight line plots lend credence to the quadratic extrapolation procedure used for Figure 4.

Laboratory Tool

The CYRES FEM modeling software. has been predominately used as a laboratory tool for mode identification and analysis. Current laboratory research procedures for research resonators make full use of the software package. Use of the software requires the material properties and dimensions of the radial cross section be written in an input file. The PC based pre-processor generates a mesh of specified density defining the resonator and its boundary conditions. The mesh information feeds the FEM processor running on a Sun workstation or Cray supercomputer. Output for each azimuthal mode number includes a list of cavity resonant frequencies and wall loss Q's, and a file. of vector magnetic field value.s. The field file. is post-processed on a PC to graphically display the magnetic and electric field vectors in three different resonator cross sections. The field visualization is used to label the modes in the. software's list solutions.

In the laboratory, modes are found by sweeping the input frequency and logging the frequency, Q, and coupling coefficient for each resonant mode.. This list is then preliminarily matched by frequency with the finite element data. Because of the arrangement of our coupling ports it is not possible to experimentally find all of the modes indicated by the FEM calculations. Analysis of the electromagnetic visualization of the resonator cross section identifies the experimental modes for the geometry. Experimentally, high Q (>5 million) whispering gallery modes arc found to have. weak coupling to the output port due. to the low field strength near the can wall. Low Q mode.s are often found to be strongly coupled to the output port, thereby indicating a relatively large field strength at the can wall. A strongly coupled high Q mode indicates a well behaved first order high azimuthal number whispering gallery mode which can reach design Q's (>12 million) when properly (critically or slightly weakly) coupled to the output port.

Using this information, the modes are identified and grouped into whispering gallery mode families by comparing the. characteristics crf the measured resonant modes m the those of the FEM modes.

As a laboratory analysis tool the software can suggest performance deficiencies of a particular mode, indicate possible design improveme nts, and identify possible. mode competition.

Design Tool

More important than the software.'s ability to model existing resonators, m a y be its design capabilities for new resonators. The FEM solution's have shown to properly and accurate.]y model every resonator we have constructed. It is expected that the software's performance will be similar for new resonator designs of cylindrical symmetry in dielectric and geometry.

New resonator geometries are being developed for improved performance. including enhanced Q's [8], stability, coupling control and, spurious mode suppression. The software. can model the. performance of the actual rc.senator. In this way, the effects of geometrical modifications on the resonators' performance can be tested without construction of experimental units. A well confined, high Q mode can be chosen around which the resonator system will be designed. The appropriate cavity for low wall losses and proper coupling can also be designed in the process. Other modes near the design frequency can be studied and selected for suppression or use as diagnostic signals.

New materials can also be implemented if they have at most a two dime.nsiona] anisotropy. Also the consequences of introducing tuning elements, probes or other perturbations can be studied. Another very important use is qualifying the predictions of the many qualitative models used for preliminary designs of resonators. For example, the software may show that the capacitive mode.1 used to design many of our resonator tuning elem ents improperly predicts the magnitude of the tuning in a new design.

Conclusion

The accuracy of the CYRES 2D FEM software methodology has been validated by comparisons with analytical and experimental results. The software has prove.n 10 be an invaluable 1001 in the laboratory for mode identification and analysis. These results demonstrate the. accuracy of the CYRES 2D software as an design tool for optimization of new rc.senators.

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